Completeness of Iris-Based Program Logics

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What is Completeness?

let $x :=$ alloc 40 in
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$!x$

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$\{\mathsf{True}\}$

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Syntactic

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What we actually want is:

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- If e terminates, φ holds
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Syntactic —Soundness→ Semantic

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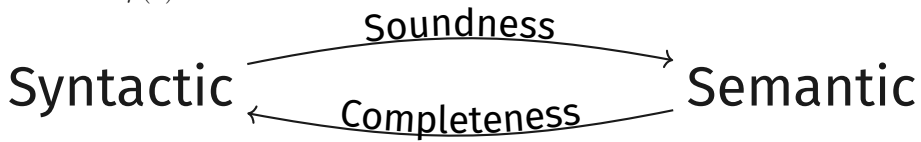

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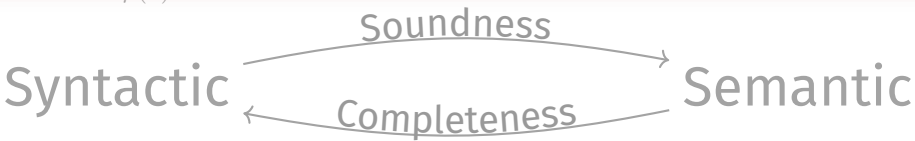
Completeness: All Correct Programs Can Be Verified



A Note On Gödel

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Proof system is not entirely syntactic!

Motivation: Resource Rules And Perfect Abstractions

In our simple example language:

$\{\ell \mapsto v\}\ !\ell\ \{w.\ulcorner w = v \urcorner * \ell \mapsto v\}$ is complete for:

$$\frac{\sigma(\ell) = v}{\langle !\ell, \sigma\rangle \to_b \langle v, \sigma\rangle}$$

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Program Logic is **Perfect Abstraction** over the Semantics!

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Iris Logic

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- ML-like lambda calculus

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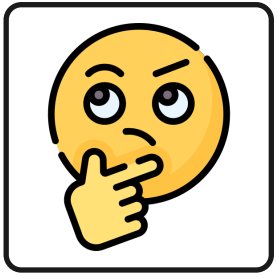
Goal will be: $\{\ulcorner \mathit{safe}_{\varphi}(e)\urcorner\}\ e\ \{v.\ \ulcorner\varphi(v)\urcorner\}$

Outline

Review the Laws

Do the Proof

Workshop the Proof



Reflect on the Proof

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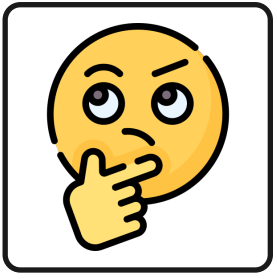

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using ~22 assertion logic rules (not shown here)

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language-specific rules

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structural rules

Value, Weaken, Frame

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Brace Yourselves – We Do the Proof!

The Proof, Part 1

$$\{\ulcorner \mathit{safe}_{\varphi}(e)\urcorner\}$$

$$e$$

$$\{v.\ulcorner\varphi(v)\urcorner\}$$

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$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_{\varphi}(e, \sigma) \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

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- Generalize over state σ

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- Generalize over state σ
- Analyze $\mathit{safe}_{\varphi}(e, \sigma)$:

$\mathit{safe}_{\varphi}(e, \sigma) \cong e$ is value satisfying φ

$\vee$ $\langle e, \sigma\rangle$ reducible

$\wedge\ \forall e', \sigma'.\ \langle e, \sigma\rangle \to \langle e', \sigma'\rangle \Rightarrow \mathit{safe}_{\varphi}(e', \sigma')$

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- Generalize over state σ
- Analyze $\mathit{safe}_{\varphi}(e, \sigma)$:
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- Generalize over state σ
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- Generalize over state σ
- Analyze $\mathit{safe}_\varphi(e, \sigma)$:
  e is value or reducible
- If e is some value w:
  From $\mathit{safe}_\varphi(w, \sigma)$: $\varphi(w)$ holds

$\mathit{safe}_\varphi(e, \sigma) \cong e$ is value satisfying φ

$\quad \vee \langle e, \sigma \rangle$ reducible

$\quad\quad \wedge \forall e', \sigma'. \langle e, \sigma \rangle \to \langle e', \sigma' \rangle \Rightarrow \mathit{safe}_\varphi(e', \sigma')$

The Proof, Part 1

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_{\varphi}(w, \sigma) \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

$$w$$

$$\{v. \ulcorner \varphi(v) \urcorner\}$$

- Generalize over state σ
- Analyze $\mathit{safe}_{\varphi}(e, \sigma)$:
  e is value or reducible
- If e is some value w:
  From $\mathit{safe}_{\varphi}(w, \sigma)$: $\varphi(w)$ holds

The Proof, Part 1

$$\ulcorner \mathit{safe}_{\varphi}(w, \sigma) \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \\ \vdash \\ \ulcorner \varphi(w) \urcorner$$

- Generalize over state σ
- Analyze $\mathit{safe}_{\varphi}(e, \sigma)$:
  e is value or reducible
- If e is some value w:
  From $\mathit{safe}_{\varphi}(w, \sigma)$: $\varphi(w)$ holds
  Apply VALUE, WEAKEN rules

The Proof, Part 1

$$\ulcorner \mathit{safe}_{\varphi}(w, \sigma) \urcorner * * _{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \vdash \ulcorner \varphi(w) \urcorner$$

$$\frac{\psi \text{ holds in meta-logic}}{P \vdash \ulcorner \psi \urcorner}$$

- Generalize over state σ
- Analyze $\mathit{safe}_{\varphi}(e, \sigma)$:
  e is value or reducible
- If e is some value w:
  From $\mathit{safe}_{\varphi}(w, \sigma)$: $\varphi(w)$ holds
  Apply VALUE, WEAKEN rules
  Conclude, as $\varphi(w)$ holds!

The Proof, Part 1

$$\left\{\begin{array}{l} \ulcorner \mathit{safe}_{\varphi}(e, \sigma) \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array}\right\}$$

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- Generalize over state σ
- Analyze $\mathit{safe}_{\varphi}(e, \sigma)$:
  e is value or reducible
- If e is some value w:
  From $\mathit{safe}_{\varphi}(w, \sigma)$: $\varphi(w)$ holds
  Apply VALUE, WEAKEN rules
  Conclude, as $\varphi(w)$ holds!
- If $\langle e, \sigma \rangle$ is reducible…

The Proof, Part 2

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_{\varphi}(e, \sigma) \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

$$e$$

$$\{v. \ulcorner \varphi(v) \urcorner\}$$

- If $\langle e, \sigma \rangle$ is reducible

The Proof, Part 2

$$\left\{\begin{array}{l} \ulcorner \mathit{safe}_{\varphi}(e, \sigma) \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array}\right\}$$

$$e$$

$$\{v. \ulcorner \varphi(v) \urcorner\}$$

- If $\langle e, \sigma \rangle$ is reducible

$$\frac{}{\langle e, \sigma \rangle \to \langle e', \sigma' \rangle}$$

The Proof, Part 2

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_{\varphi}(K[e], \sigma) \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

$$K[e]$$

$$\{v. \ulcorner \varphi(v) \urcorner\}$$

- If $\langle K[e], \sigma \rangle$ is reducible

$$\frac{\langle e, \sigma \rangle \rightarrow_b \langle e', \sigma' \rangle}{\langle K[e], \sigma \rangle \rightarrow \langle K[e'], \sigma' \rangle}$$

The Proof, Part 2

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[e], \sigma)\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$
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$\mathit{safe}_{\varphi}(e, \sigma) \cong e$ is value satisfying φ
$\vee$ $\langle e, \sigma\rangle$ reducible
$\wedge\ \forall e', \sigma'.\ \langle e, \sigma\rangle \to \langle e', \sigma'\rangle \Rightarrow \mathit{safe}_{\varphi}(e', \sigma')$

The Proof, Part 2

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$$K[e]$$
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- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

$\mathit{safe}_{\varphi}(e, \sigma) \cong e$ is value satisfying φ
$\vee\ \langle e, \sigma\rangle$ reducible
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$\mathit{safe}_{\varphi}(e, \sigma) \cong e$ is value satisfying φ

$\vee\ \langle e, \sigma\rangle$ reducible

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We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

The Proof, Part 2

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[e'], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\, \ell \mapsto v\end{array}\right\}$$

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We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

The Proof, Part 2

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[v], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$

$$K[!\ell]$$

$$\{v.\ \ulcorner \varphi(v) \urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = !\ell$: $\sigma(\ell) = v,\ e' = v, \sigma' = \sigma$

The Proof, Part 2

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[v], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$

$$K[!\ell]$$

$$\{v.\ \ulcorner \varphi(v) \urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = !\ell$: $\sigma(\ell) = v,\ e' = v, \sigma' = \sigma$

The Proof, Part 2

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_\varphi(K[v], \sigma') \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

$$!\ell$$

$$\{w.\ \{\mathsf{True}\}\ K[w]\ \{v.\ \ulcorner \varphi(v) \urcorner\}\}$$

- If $\langle K[e], \sigma \rangle$ is reducible

$$\frac{\langle e, \sigma \rangle \to_b \langle e', \sigma' \rangle}{\langle K[e], \sigma \rangle \to \langle K[e'], \sigma' \rangle}$$

We know $\mathit{safe}_\varphi(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = !\ell$: $\sigma(\ell) = v,\ e' = v, \sigma' = \sigma$

  Apply BIND rule

The Proof, Part 2

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_\varphi(K[v], \sigma') \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

$!\ell$

$$\{w.\, \{\mathsf{True}\}\ K[w]\ \{v.\, \ulcorner \varphi(v) \urcorner\}\}$$

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  Apply BIND rule

The Proof, Part 2

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[v], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$

$$K[v]$$

$$\{v.\ulcorner \varphi(v)\urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = !\ell$: $\sigma(\ell) = v,\ e' = v, \sigma' = \sigma$

  Apply BIND rule

  Apply LOAD rule

The Proof, Part 2

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[v], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$

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Case distinction on $\to_b$:

- Case $e = !\ell$: $\sigma(\ell) = v,\ e' = v, \sigma' = \sigma$

  Apply BIND rule

  Apply LOAD rule

  Observe: we are back where we started!

The Proof, Part 2

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[v], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$

$$K[v]$$

$$\{v.\ \ulcorner \varphi(v) \urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = !\ell$: $\sigma(\ell) = v,\ e' = v, \sigma' = \sigma$

  Apply BIND rule — Observe: we are back where we started!

  Apply LOAD rule — Conclude by induction on steps.

  (assuming termination, for now)

The Proof, Part 3

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[e'], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\, \ell \mapsto v\end{array}\right\}$$

$$K[e]$$

$$\{v.\ulcorner \varphi(v)\urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

The Proof, Part 3

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[e_1[v/x]], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$

$$K[\text{let } x := v \text{ in } e_1]$$

$$\{v.\ \ulcorner \varphi(v) \urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = \text{let } x := v \text{ in } e_1$: $e' = e_1[v/x], \sigma' = \sigma$

The Proof, Part 3

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_\varphi(K[e_1[v/x]], \sigma') \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

let $x := v$ in e_1

$$\{w.\ \{\mathsf{True}\}\ K[w]\ \{v.\ \ulcorner \varphi(v) \urcorner\}\}$$

- If $\langle K[e], \sigma \rangle$ is reducible

$$\frac{\langle e, \sigma \rangle \to_b \langle e', \sigma' \rangle}{\langle K[e], \sigma \rangle \to \langle K[e'], \sigma' \rangle}$$

We know $\mathit{safe}_\varphi(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e =$ let $x := v$ in e_1: $e' = e_1[v/x], \sigma' = \sigma$

  Apply BIND rule

The Proof, Part 3

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_\varphi(K[e_1[v/x]], \sigma') \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

$e_1[v/x]$

$$\{w.\, \{\mathsf{True}\}\ K[w]\ \{v. \ulcorner \varphi(v) \urcorner\}\}$$

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$$\frac{\langle e, \sigma \rangle \to_b \langle e', \sigma' \rangle}{\langle K[e], \sigma \rangle \to \langle K[e'], \sigma' \rangle}$$

We know $\mathit{safe}_\varphi(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = \mathsf{let}\ x := v\ \mathsf{in}\ e_1$: $e' = e_1[v/x], \sigma' = \sigma$

  Apply Bind rule

  Apply Let rule

The Proof, Part 3

$$\left\{ \begin{array}{l} \ulcorner \mathit{safe}_\varphi(K[e_1[v/x]], \sigma') \urcorner * \\ *_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \end{array} \right\}$$

$$e_1[v/x]$$

$$\{w.\, \{\mathsf{True}\}\ K[w]\ \{v. \ulcorner \varphi(v) \urcorner\}\}$$

- If $\langle K[e], \sigma \rangle$ is reducible

$$\frac{\langle e, \sigma \rangle \to_b \langle e', \sigma' \rangle}{\langle K[e], \sigma \rangle \to \langle K[e'], \sigma' \rangle}$$

We know $\mathit{safe}_\varphi(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = \mathsf{let}\ x := v\ \mathsf{in}\ e_1$: $e' = e_1[v/x], \sigma' = \sigma$

  Apply Bind rule

  Apply Let rule

$$\frac{\{P\}\ e\ \{v.\, \{\mathsf{True}\}\ K[v]\ \{Q\}\}}{\{P\}\ K[e]\ \{Q\}}\ \textsc{Bind}$$

The Proof, Part 3

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_\varphi(K[e_1[v/x]], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma}\ \ell \mapsto v\end{array}\right\}$$

$$K[e_1[v/x]]$$

$$\{v.\ \ulcorner \varphi(v) \urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_\varphi(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = \text{let } x := v \text{ in } e_1$: $e' = e_1[v/x], \sigma' = \sigma$

  Apply Bind rule

  Apply Let rule

  Apply Bind rule in reverse

$$\frac{\{P\}\ e\ \{v.\ \{\text{True}\}\ K[v]\ \{Q\}\}}{\{P\}\ K[e]\ \{Q\}}\ \textsc{Bind}$$

The Proof, Part 3

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[e_1[v/x]], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma'} \ell \mapsto v\end{array}\right\}$$

$$K[e_1[v/x]]$$

$$\{v.\ulcorner \varphi(v)\urcorner\}$$

- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = \text{let } x := v \text{ in } e_1$: $e' = e_1[v/x], \sigma' = \sigma$

  Apply Bind rule

  Apply Let rule

  Apply Bind rule in reverse

The Proof, Part 3

$$\left\{\begin{array}{l}\ulcorner \mathit{safe}_{\varphi}(K[e_1[v/x]], \sigma')\urcorner * \\ *_{(\ell \leftarrow v) \in \sigma'} \ell \mapsto v\end{array}\right\}$$

$$K[e_1[v/x]]$$

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- If $\langle K[e], \sigma\rangle$ is reducible

$$\frac{\langle e, \sigma\rangle \to_b \langle e', \sigma'\rangle}{\langle K[e], \sigma\rangle \to \langle K[e'], \sigma'\rangle}$$

We know $\mathit{safe}_{\varphi}(K[e'], \sigma')$

Case distinction on $\to_b$:

- Case $e = \text{let } x := v \text{ in } e_1$: $e' = e_1[v/x], \sigma' = \sigma$
  - Apply BIND rule — We are back where we started!
  - Apply LET rule
  - Apply BIND rule in reverse

We did it!

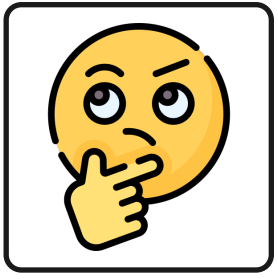
We did it!

Outline

Review the Laws

Do the Proof

Workshop the Proof



Reflect on the Proof

The Proof, Summary

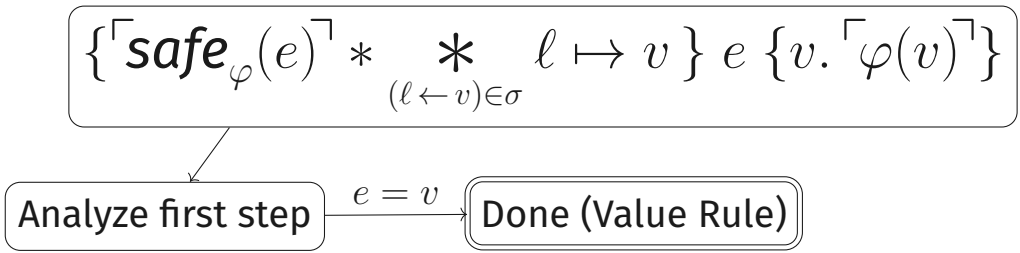
$$\{\ulcorner \mathit{safe}_{\varphi}(e) \urcorner * \mathop{\Large *}_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v \}\ e\ \{v. \ulcorner \varphi(v) \urcorner\}$$

The Proof, Summary

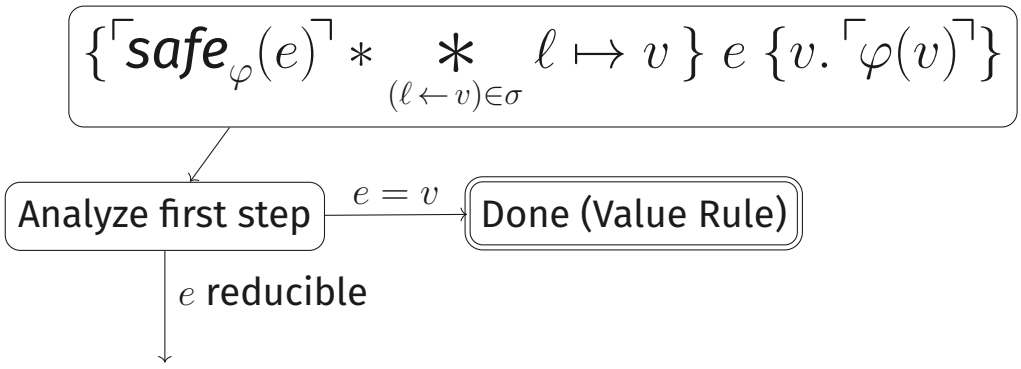
$$\{\ulcorner \mathit{safe}_{\varphi}(e) \urcorner * \mathop{\Large *}_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v\}\ e\ \{v.\ \ulcorner \varphi(v) \urcorner\}$$

Analyze first step

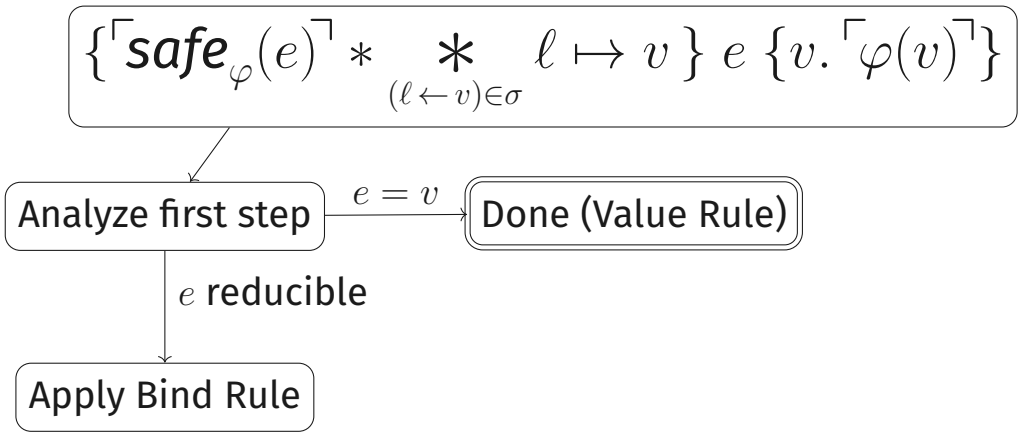
The Proof, Summary



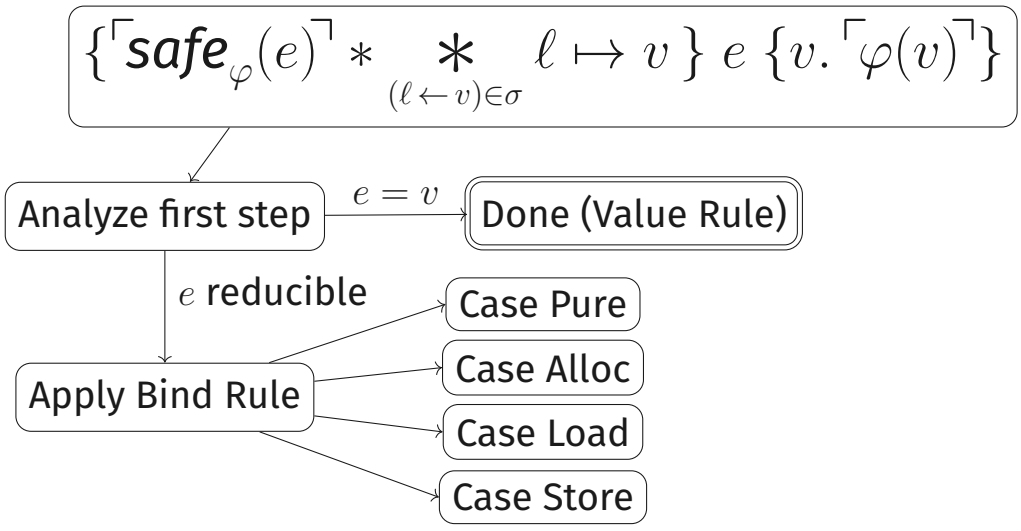
The Proof, Summary



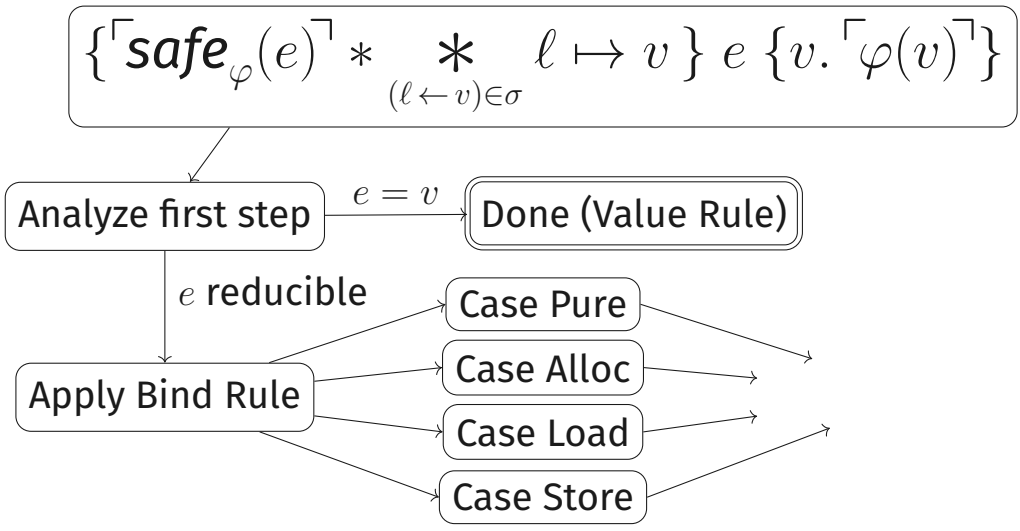
The Proof, Summary



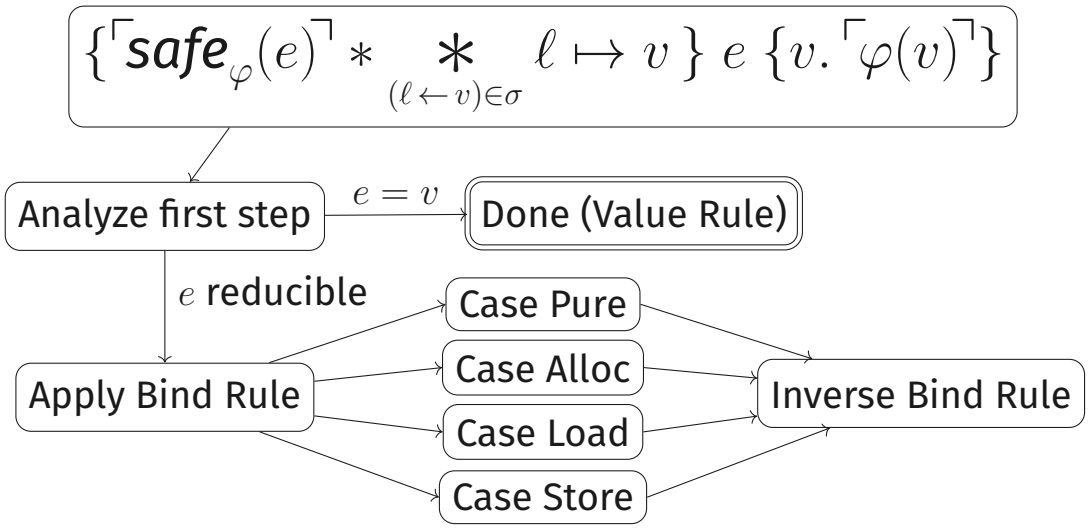
The Proof, Summary



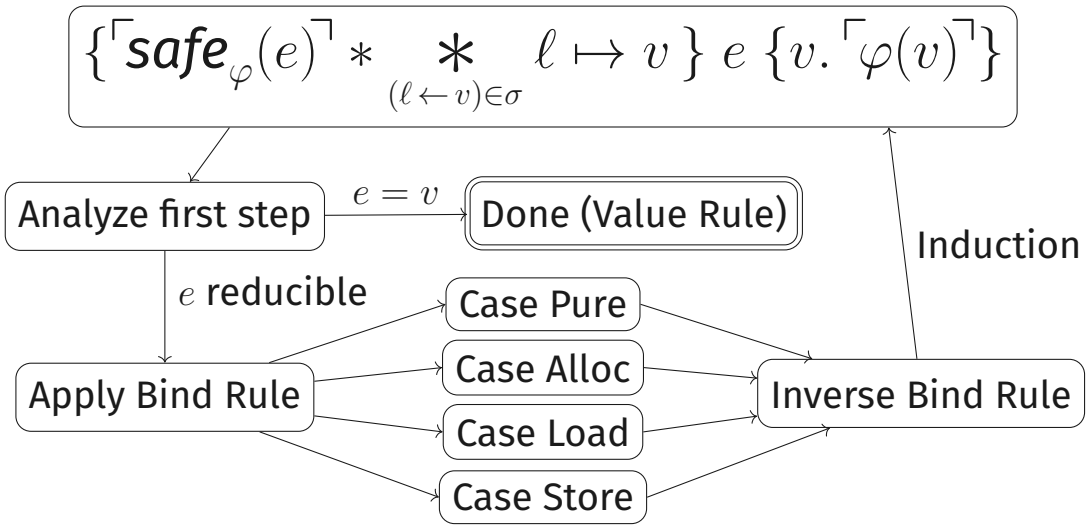
The Proof, Summary



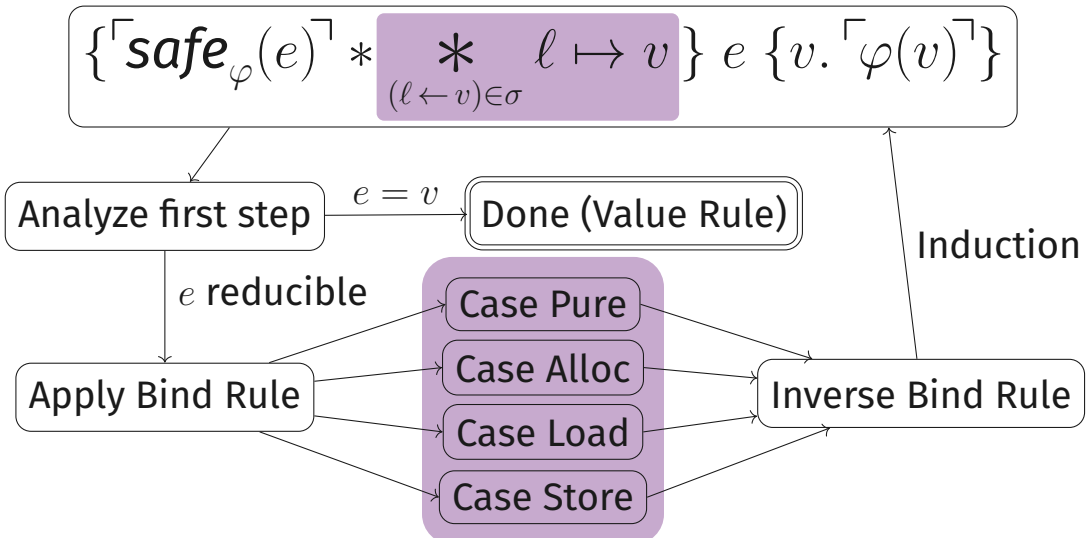
The Proof, Summary



The Proof, Summary



The Proof, Summary and Generalizations



Strengthenings

So far: Completeness for sequential, terminating programs.

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Non-termination:

Strengthenings

So far: Completeness for sequential, terminating programs.

Non-termination: safe is co-inductive.

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Solution: Hoare triples are also co-inductive.

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So far: Completeness for sequential, terminating programs.

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Solution: Hoare triples are ~~also co-inductive~~ *step-indexed*.

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So far: Completeness for sequential, terminating programs.

Non-termination: safe is co-inductive.

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Concurrency:

Strengthenings

So far: Completeness for sequential, terminating programs.

Non-termination: safe is co-inductive.

Solution: Hoare triples are ~~also co-inductive~~ *step-indexed*.

Concurrency: We must share resources among threads.

Strengthenings

So far: Completeness for sequential, terminating programs.

Non-termination: safe is co-inductive.

Solution: Hoare triples are ~~also co-inductive~~ *step-indexed*.

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Solution: Invariants

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Solution: Invariants

$$\exists \vec{e}, \sigma.\ \ulcorner \text{safe-tp}_{\varphi}(\vec{e}, \sigma) \urcorner * \bullet^{\gamma} \vec{e} * \ast_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v$$

Strengthenings

So far: Completeness for sequential, terminating programs.

Non-termination: safe is co-inductive.

Solution: Hoare triples are ~~also co-inductive~~ *step-indexed*.

Concurrency: We must share resources among threads.

Solution: Invariants and more technical difficulties…

$$\exists \vec{e}, \sigma.\ \ulcorner \mathsf{safe\text{-}tp}_{\varphi}(\vec{e}, \sigma) \urcorner * \bullet^{\gamma} \vec{e} * \ast_{(\ell \leftarrow v) \in \sigma} \ell \mapsto v$$

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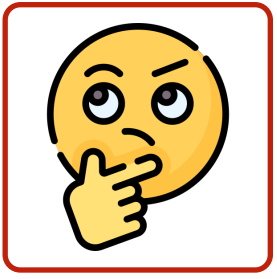
Do the Proof

Workshop the Proof



Reflect on the Proof

Discussion: Case Studies

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Small incompletenesses found and fixed!

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- Later credits, > 1 laters per step, Transfinite Iris

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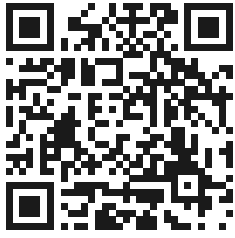
The End

Thanks for your attention!

More info:



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